All pure multipartite entangled states of qubits can be self-tested

Ivan Šupić
CNRS, Laboratoire d'Informatique de Grenoble

QIP2026 , Riga, July 29th 2026

Joint work with:



Maria Balanzo-Juando
ULB
Brussels



Andrea Coladangelo
University of Washington
Seattle



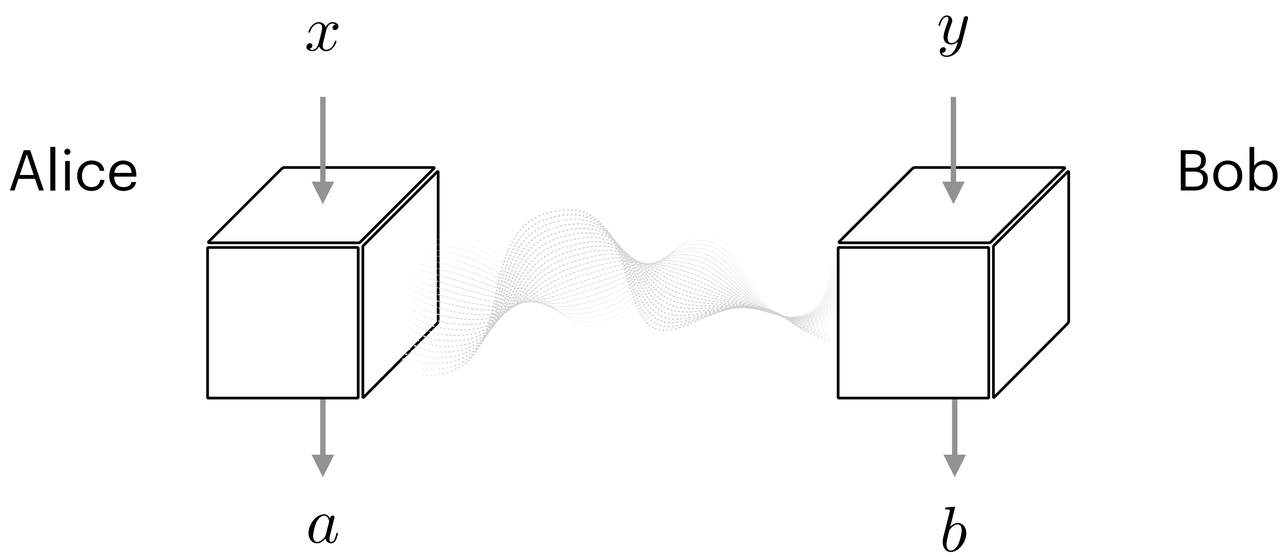
Remigiusz Augusiak
Center for Theoretical Physics
Warsaw



Antonio Acin
ICFO
Barcelona

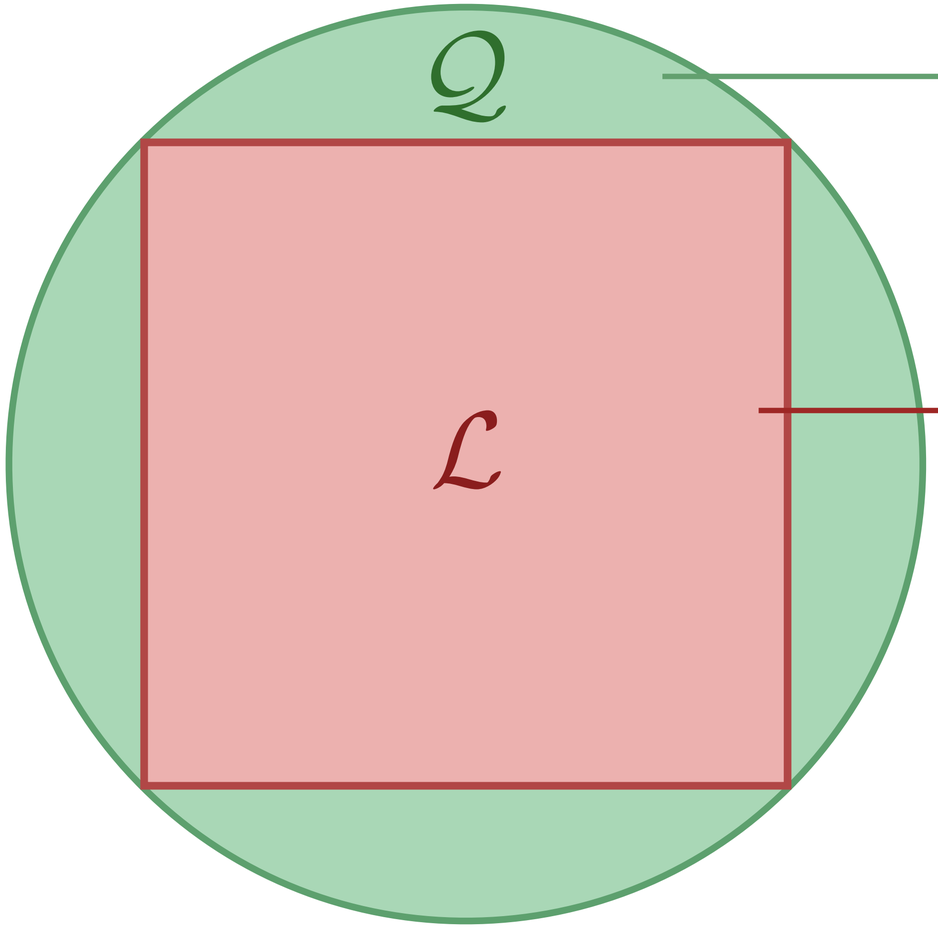
All pure multipartite qubit states can be self-tested up to complex conjugation
M. Balazo-Juando, A. Coladangelo, R. Augusiak, A. Acin, IŠ
arXiv: 2412.13266

(Bipartite) Bell scenario



Correlation $p(a, b|x, y)$

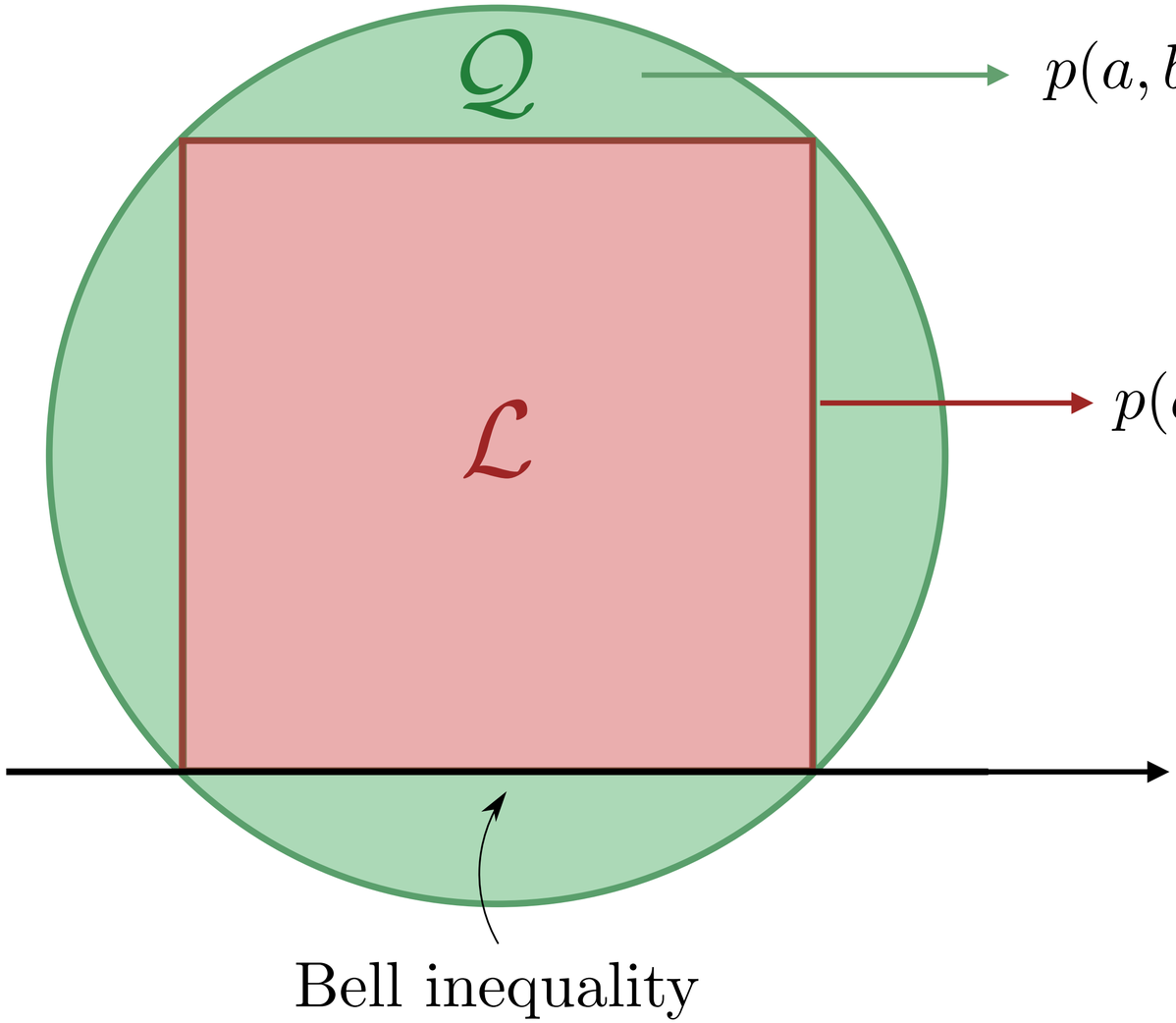
Bell inequalities



$$p(a,b|x,y) = \mathrm{tr}(\rho_{AB}\, M_{a|x} \otimes N_{b|y})$$

$$p(a,b|x,y) = \int d\lambda\, q(\lambda)\; p(a|x\lambda)\, p(b|y,\lambda)$$

Bell inequalities



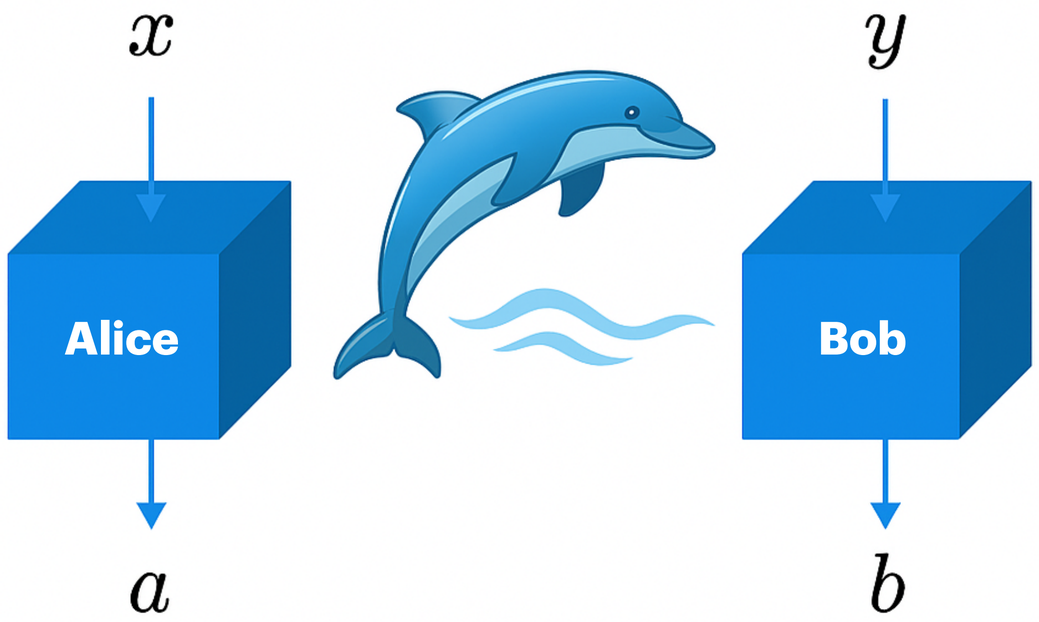
$$p(a,b|x,y) = \mathrm{tr}(\rho_{AB}\, M_{a|x} \otimes N_{b|y})$$

$$p(a,b|x,y) = \int d\lambda\, q(\lambda)\; p(a|x\lambda)\, p(b|y,\lambda)$$

$$\sum_{abxy} w_{abxy}\, p(a,b|x,y) \leq \eta$$

Device-independent formalism

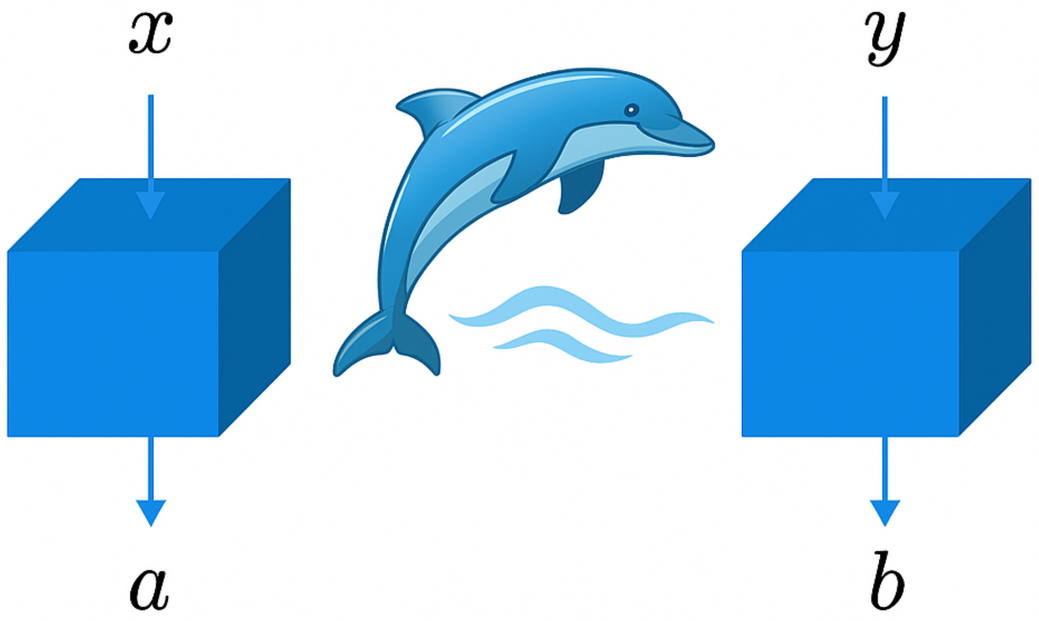
Devices treated as black boxes



Correlation $p(a, b|x, y)$

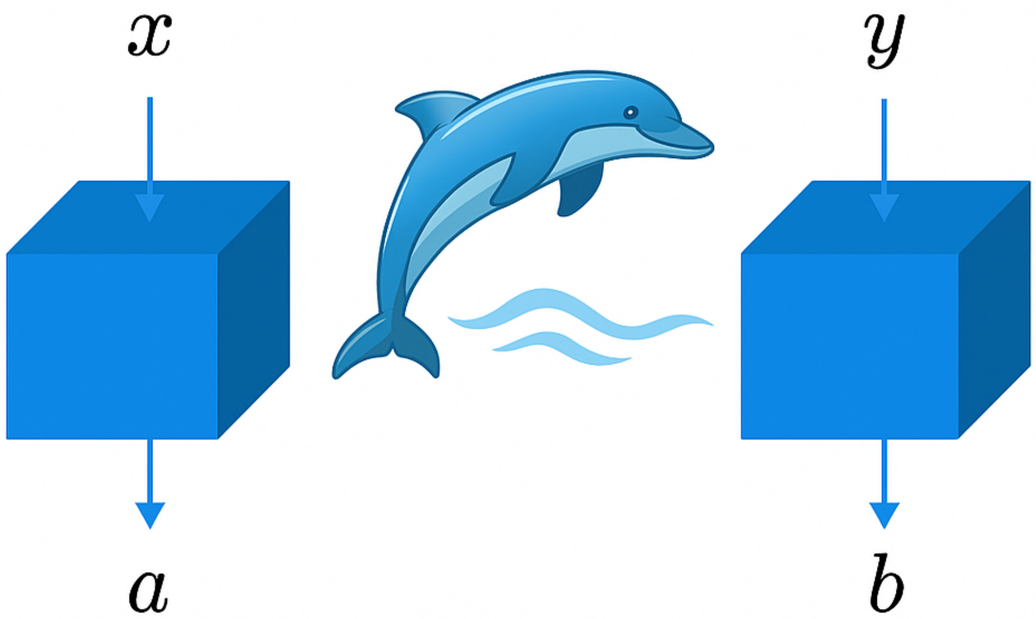
Device-independent paradigm

- Observing violation larger than η witnesses entanglement and incompatibility
- This witness is based only on the observed probability distribution $p(a, b \,|\, x, y)$
- There is no trust whatsoever in any of the devices used in the experiment
- This form of witnessing is known as **device-independent**.



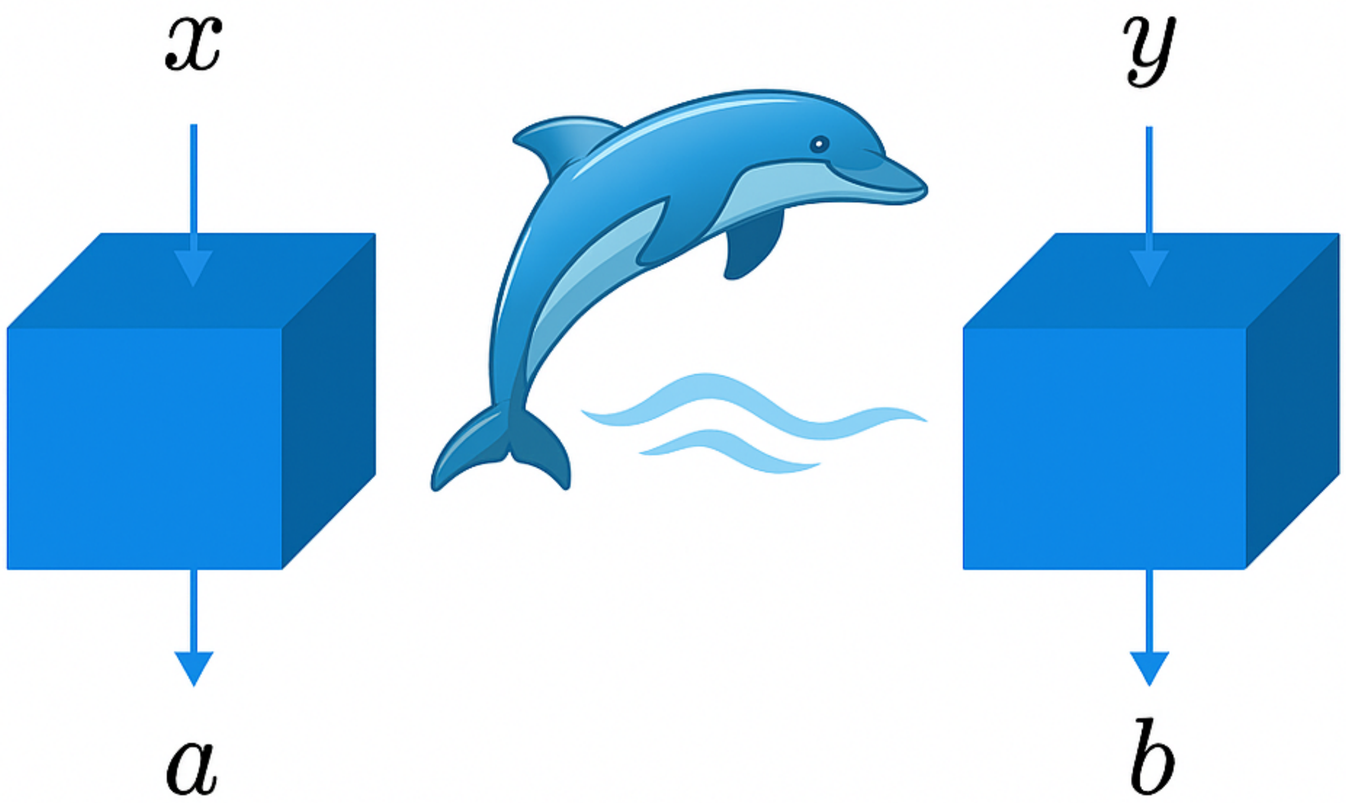
Self-testing

Strongest form of certification

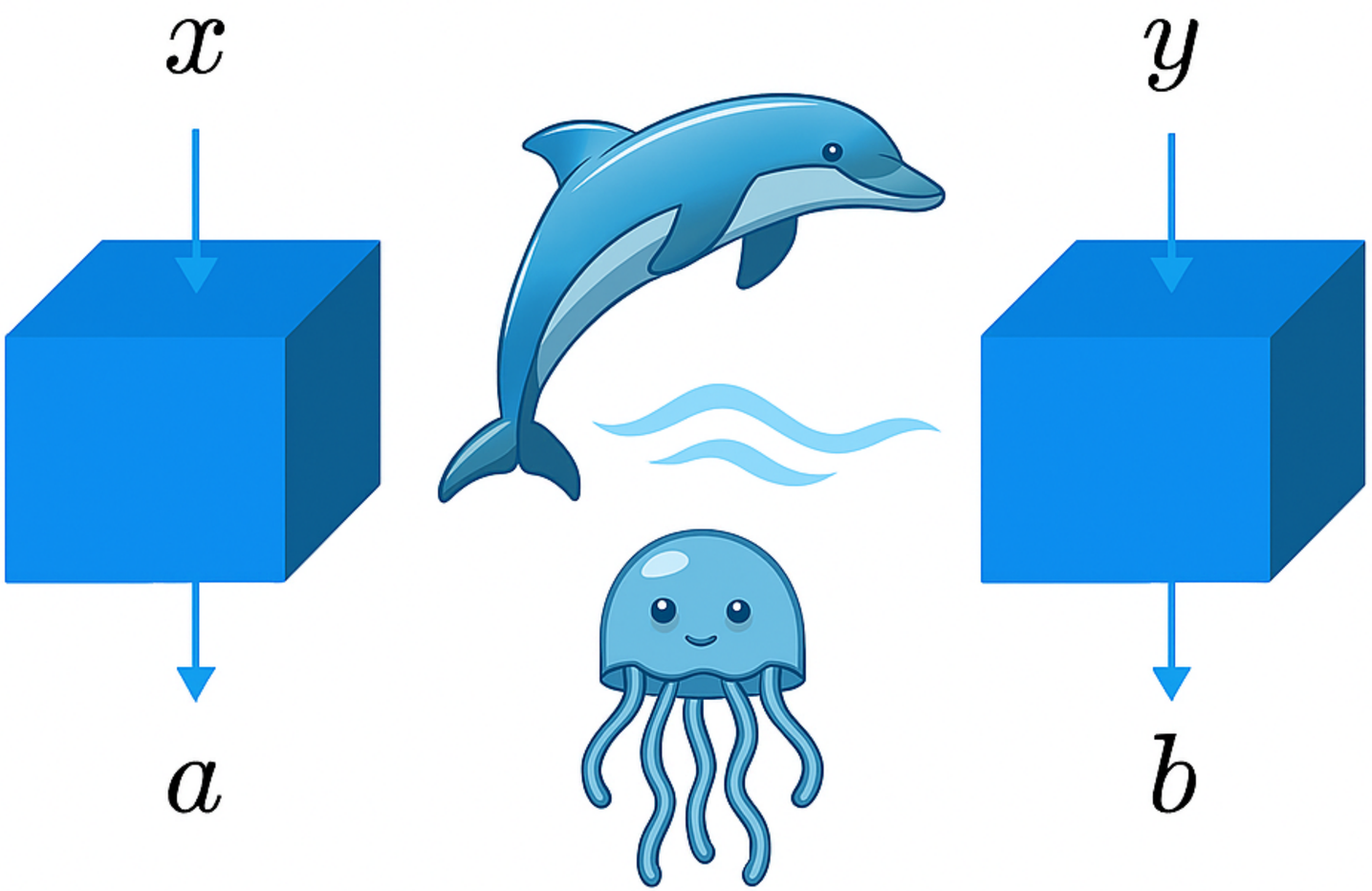


What is the shared state? What are the measurements?

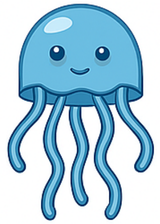
Identifying the exact state?



Identifying the exact state?

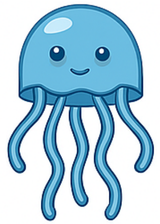


Self-testing: isometries



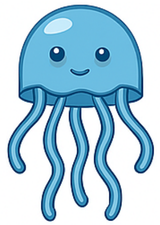
Local change of basis

$$\{U_A \otimes U_B |\psi\rangle, U_A M_{a|x} U_A^\dagger, U_B N_{b|y} U_B^\dagger\} \Leftrightarrow \{|\psi\rangle, M_{a|x}, N_{b|y}\}$$



Uncorrelated degrees of freedom to which measurements act trivially

$$\{|\psi\rangle^{AB} \otimes |\phi\rangle^{A'B'}, M^A_{a|x} \otimes \mathbb{I}^{A'}, N^B_{b|y} \otimes \mathbb{I}^{B'}\} \Leftrightarrow \{|\psi\rangle, M_{a|x}, N_{b|y}\}$$

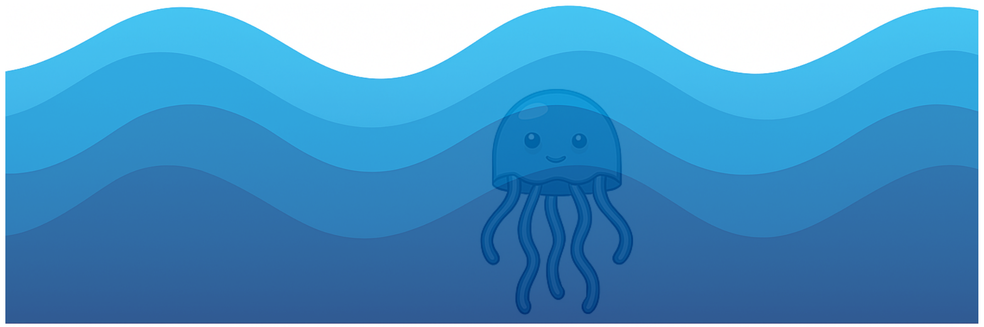


These transformations are are supposed to be encompassed by the local extraction maps

$$\tau_A \otimes \tau_B(|\psi'\rangle) \Leftrightarrow |\psi\rangle$$

Extraction map = mixed isometry

Self-testing: complex conjugation



Simultaneous complex conjugation of states and measurements does not change the correlation probabilities

$$\tau(|\psi'\rangle) = |\psi\rangle \otimes |\text{junk}_0\rangle + |\psi^*\rangle \otimes |\text{junk}_1\rangle$$

$$\langle \text{junk}_0 | \text{junk}_0 \rangle + \langle \text{junk}_1 | \text{junk}_1 \rangle = 1$$

$$\langle \text{junk}_0 | \text{junk}_1 \rangle = 0$$

Self-testing from Bell inequalities



$$|\phi_+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

$$I_{\mathrm{CHSH}} = \langle A_0 B_0 \rangle + \langle A_0 B_1 \rangle + \langle A_1 B_0 \rangle - \langle A_1 B_1 \rangle$$



$$|\phi_\theta\rangle = \cos(\theta)|00\rangle + \sin(\theta)|11\rangle$$

$$I_{\mathrm{tilted}} = \alpha\langle A_0 \rangle + \langle A_0 B_0 \rangle + \langle A_0 B_1 \rangle + \langle A_1 B_0 \rangle - \langle A_1 B_1 \rangle$$

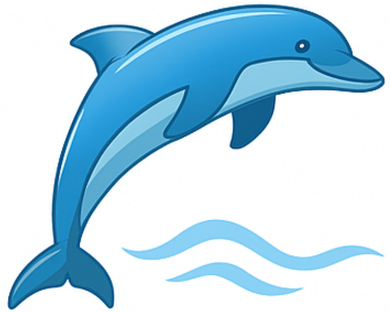
Self-testing

Known results

- Partially entangled qubits — *[Bamps, Pironio, 2015]*
- Qudit pure bipartite entangled — *[Coladangelo, Goh, Scarani, 2017]*
- Graph states — *[McKague, 2014]*
- GHZ states — *[Pál, Vértesi, Navascués, 2014]*
- Dicke states — *[Šupić, Coladangelo, Augusiak, Acín, 2018]*

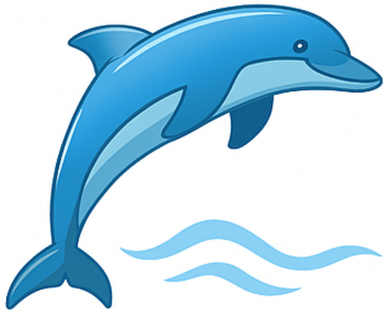
- All multipartite states? — *Quantum networks self-test all entangled states* *[Šupić, Bowles, Renou, Acín, Hoban, 2023]*

Motivation

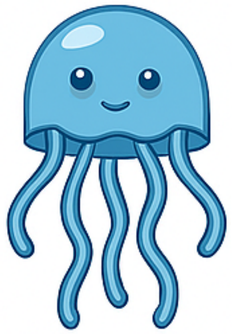


Networks solve the problem

Motivation

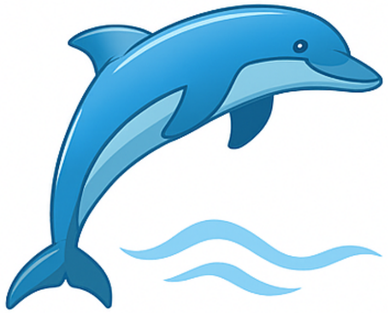


Networks solve the problem

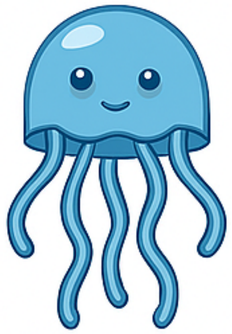


Require additional resources: N EPR pairs

Motivation



Networks solve the problem



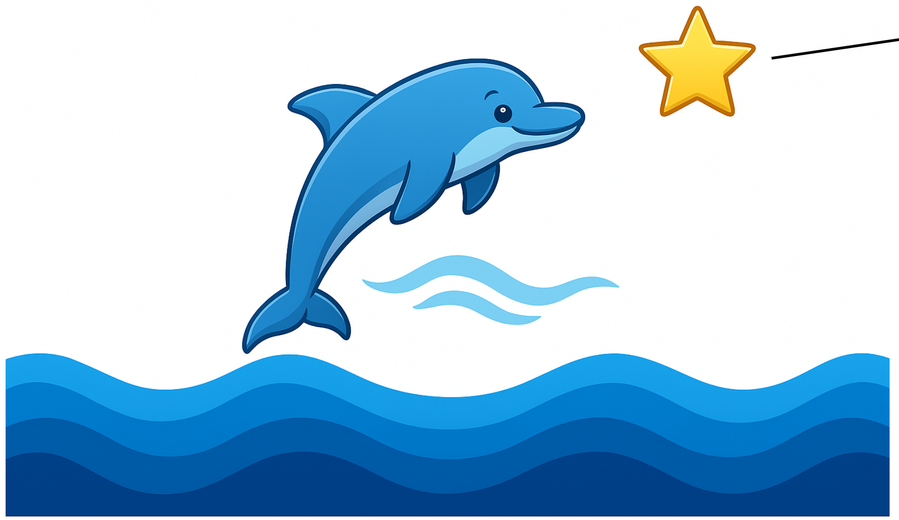
Require additional resources: N EPR pairs



What happens with self-testing
in the standard Bell scenario?

Motivation

- Only results for bipartite states and some multipartite.
- Self-test is tailored to the state, or a class of states sharing some symmetry
- Multipartite is hard → the structure of multipartite entanglement is very diverse

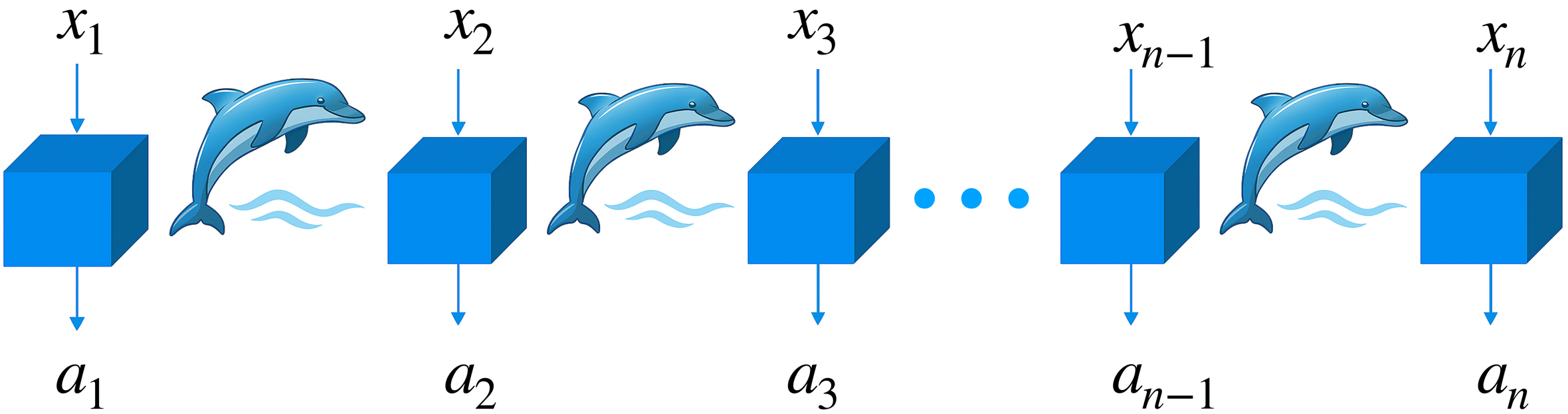


Find a general way to self-test

multipartite qubit entangled states

Scenario

Multipartite scenario



Correlation $p(a_1, \ldots, a_n | x_1 \ldots x_n)$

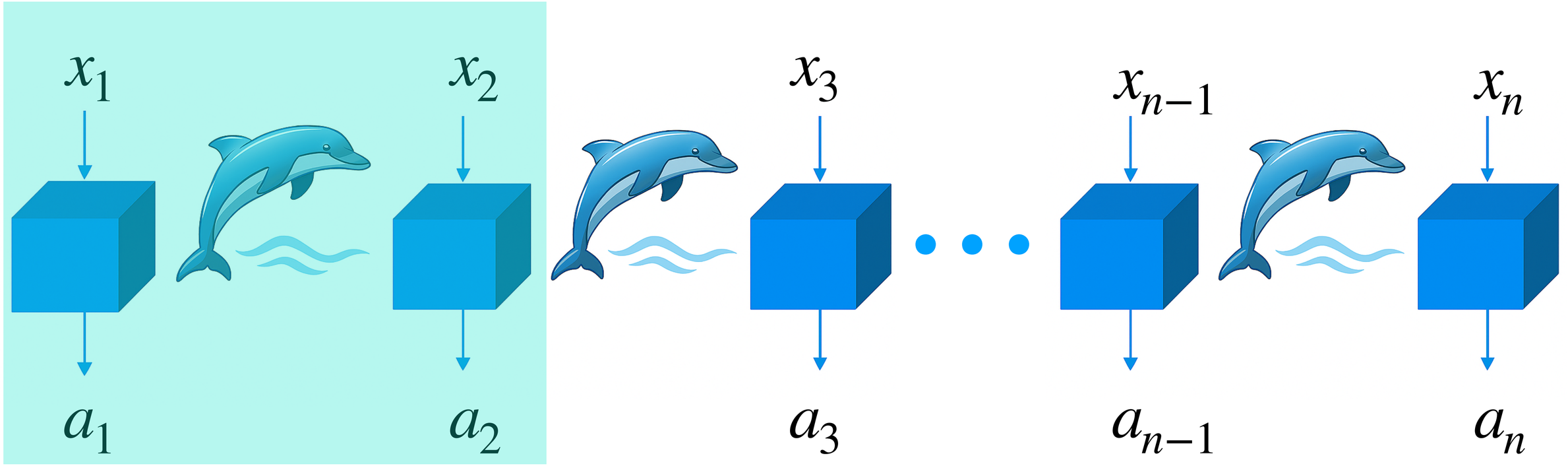
Multipartite scenario

The idea behind

Use successive self-testing of bipartite states for all possible bipartitions → different sub-tests.

To do this, project the rest of the parties + assume the bipartite state is entangled.

Self-test the bipartite entangled state and the measurements.



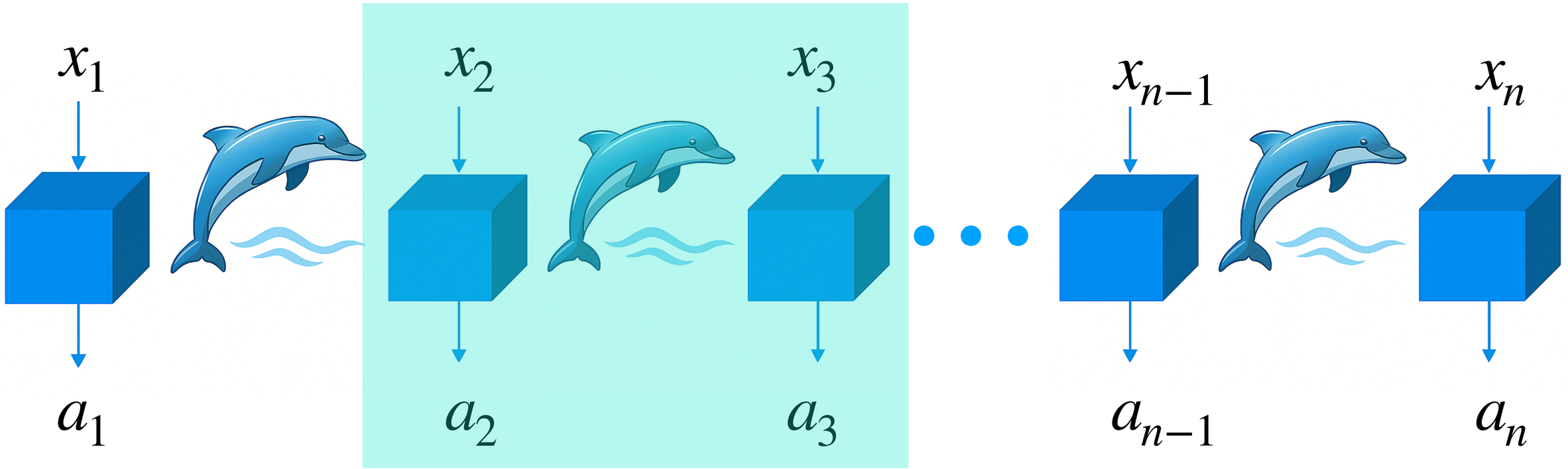
Multipartite scenario

The idea behind

Use successive self-testing of bipartite states for all possible bipartitions → different sub-tests.

To do this, project the rest of the parties + assume the bipartite state is entangled.

Self-test the bipartite entangled state and the measurements.



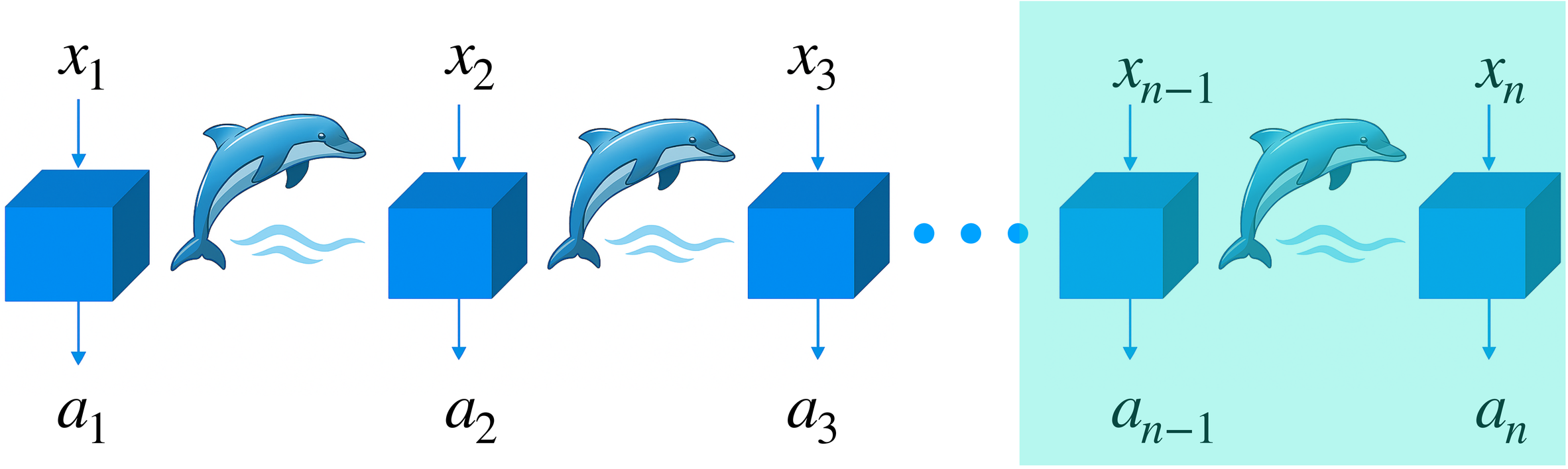
Multipartite scenario

The idea behind

Use successive self-testing of bipartite states for all possible bipartitions → different sub-tests.

To do this, project the rest of the parties + assume the bipartite state is entangled.

Self-test the bipartite entangled state and the measurements.



Tripartite states

Tripartite states

Example

Consider the target state $|\psi\rangle = |0\rangle_A \otimes |\phi^+\rangle_{BC} + |1\rangle_A \otimes |\psi^-\rangle_{BC}$

Obtain 0 → project to $|\phi^+\rangle$

→ Max violates $CHSH_1$

$$I_{CHSH_1} = \langle A_0B_0\rangle + \langle A_0B_1\rangle + \langle A_1B_0\rangle - \langle A_1B_1\rangle$$

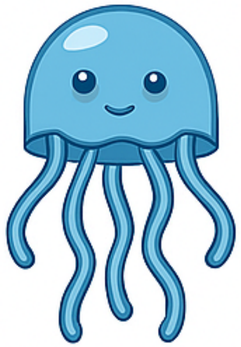
Obtain 1 → project to $|\psi^-\rangle$

→ Max violates $CHSH_4$

$$I_{CHSH_4} = -\langle A_0B_0\rangle - \langle A_0B_1\rangle - \langle A_1B_0\rangle + \langle A_1B_1\rangle$$

Tripartite states

Example

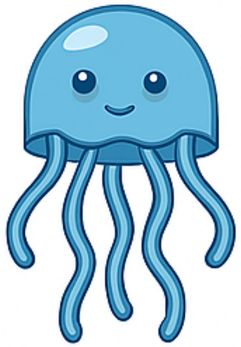


$$\rho = \frac{1}{2}|0\rangle\langle 0| \otimes |\phi^+\rangle\langle\phi^+| + \frac{1}{2}|1\rangle\langle 1| \otimes |\psi^-\rangle\langle\psi^-|$$

This state is separable across the bipartition A|BC → Not the target state.

Tripartite states

Example



$$\rho = \frac{1}{2}|0\rangle\langle 0| \otimes |\phi^+\rangle\langle\phi^+| + \frac{1}{2}|1\rangle\langle 1| \otimes |\psi^-\rangle\langle\psi^-|$$

This state is separable across the bipartition A|BC → Not the target state.



Establish that there is entanglement across A|BC.

Tripartite states

Example

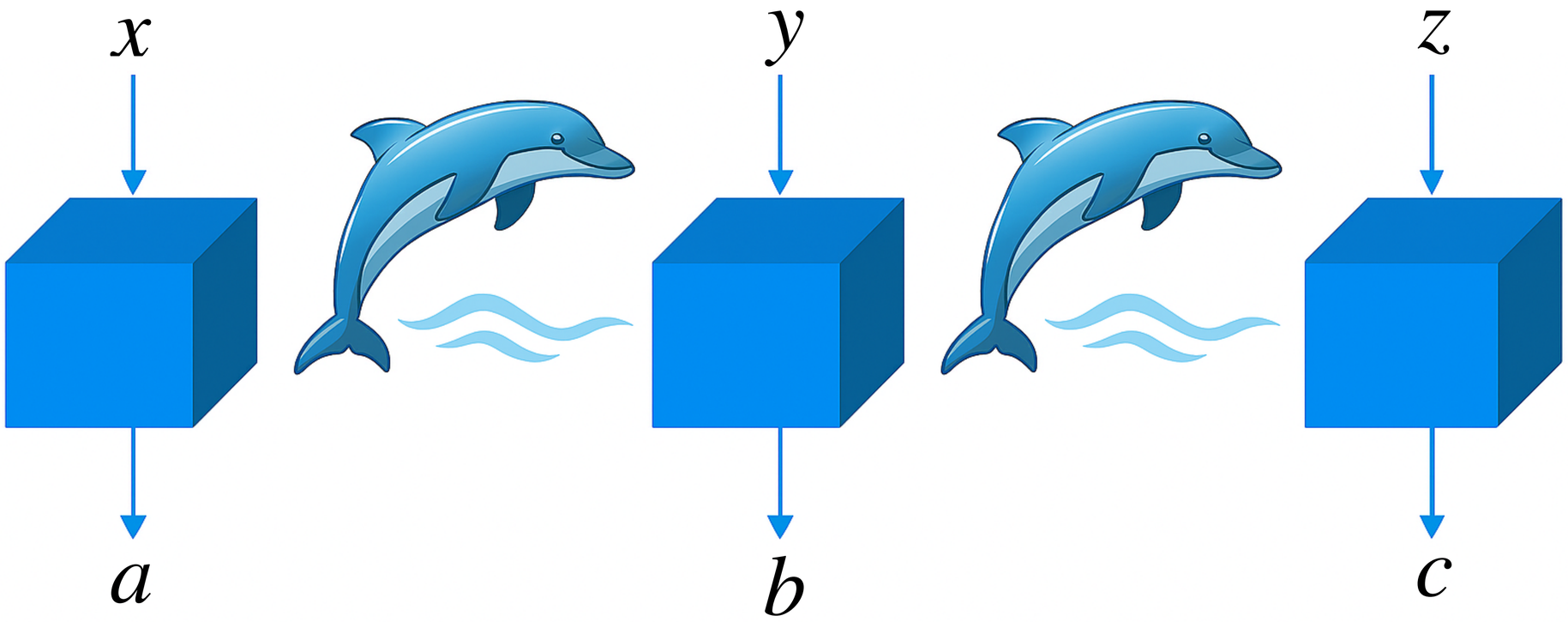
The target state can also be written as: $|\psi\rangle = |\phi^-\rangle_{AB} \otimes |0\rangle_C + |\psi^+\rangle_{AB} \otimes |1\rangle_C$

In this sub-test we can do the same: Charlie measures in the computational basis and the other parties play CHSH games.

If they win every time, there is enough information to self-test the whole state.

Tripartite states

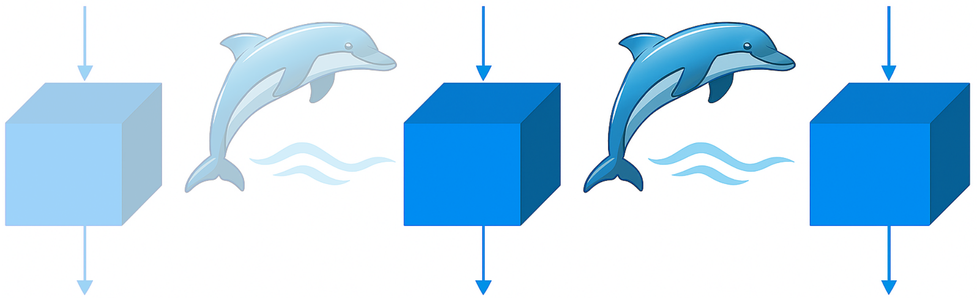
The simplest multipartite case



What is the shared state? $|\psi\rangle = \sum_{ijk} \lambda_{ijk} |ijk\rangle$

Tripartite states

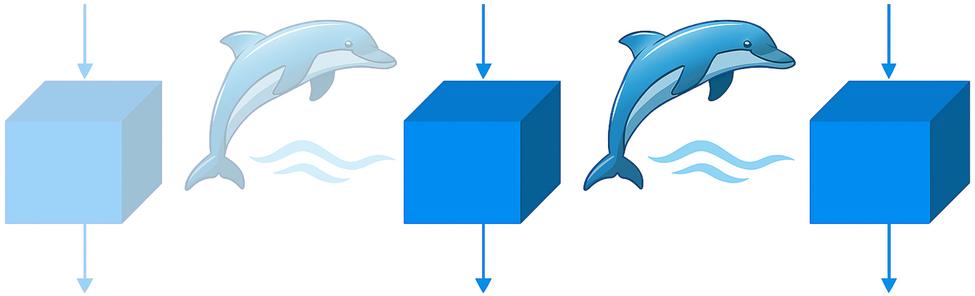
The idea behind



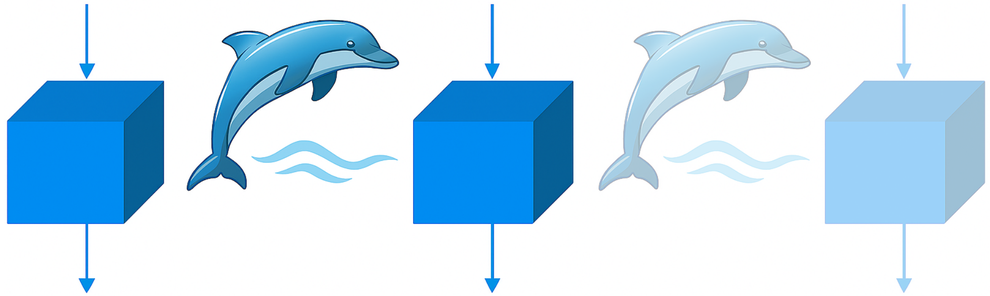
Alice projects, Bob and Charlie test

Tripartite states

The idea behind



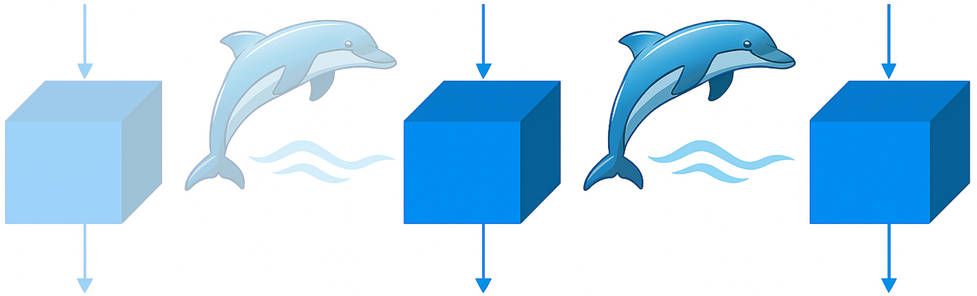
Alice projects, Bob and Charlie test



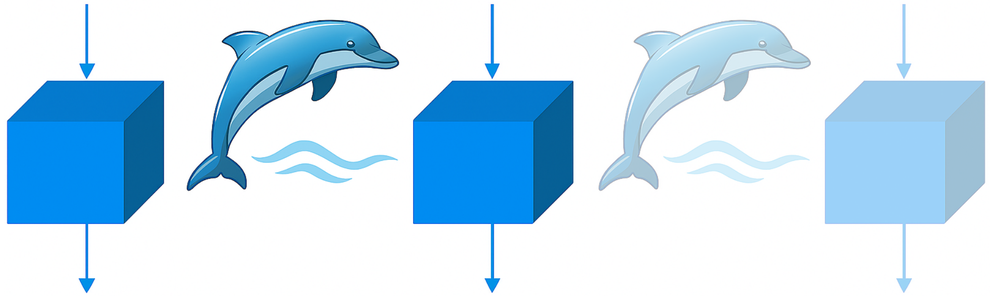
Charlie projects, Bob and Alice test

Tripartite states

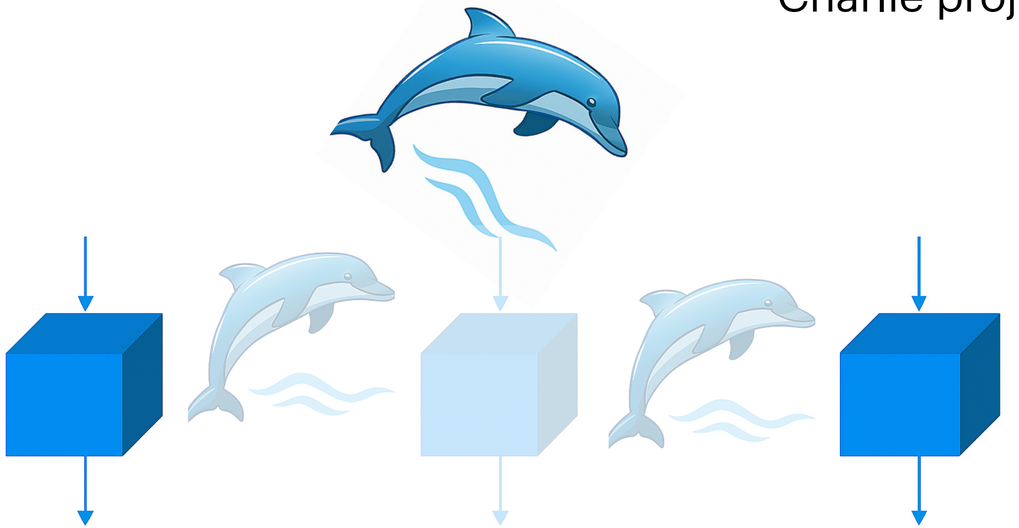
The idea behind



Alice projects, Bob and Charlie test



Charlie projects, Bob and Alice test



Bob projects, Alice and Charlie test

Tool 1: Full bipartite self-testing



Set of 3 Bell inequalities with

3 inputs on Alice’s side and

6 inputs on Bob’s side in total

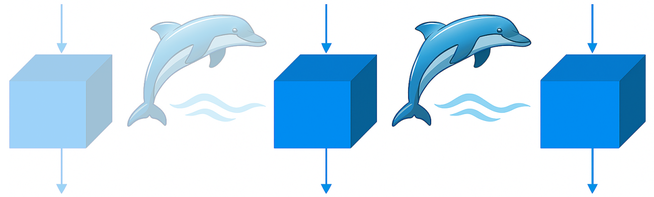
→ 2 tilted CHSH + 1 CHSH.

It can be used to self-test the **partially entangled pair of qubits** and **three Pauli observables** on Alice’s side (up to the complex conjugation uncertainty).

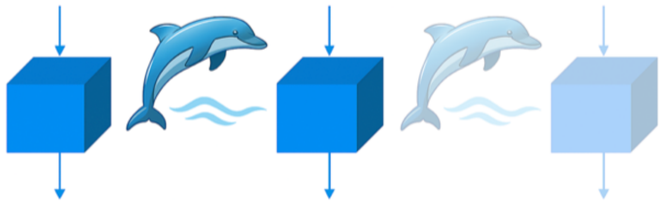
E. Woodhead, J. Kaniewski, B. Bourdoncle, A. Salavrakos, J. Bowles, R. Augusiak, and A. Acín. Phys. Rev. Research 2, 042028(R), (2020).

What do we expect?

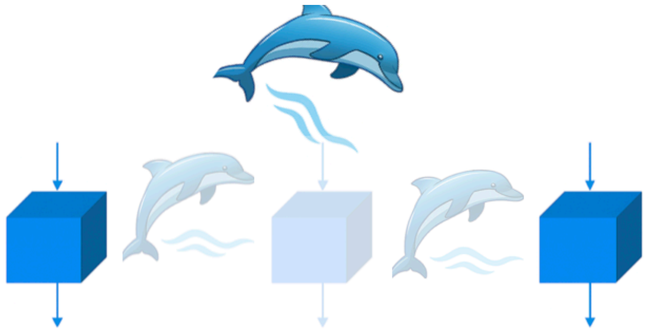
- When party A makes ***computational basis measurement*** the remaining two parties are projected to $\sum_{j,k} \lambda_{0,j,k} |jk\rangle$ and $\sum_{j,k} \lambda_{1,j,k} |jk\rangle$



- When party B makes ***computational basis measurement*** the remaining two parties are projected to $\sum_{i,k} \lambda_{i,0,k} |ik\rangle$ and $\sum_{i,k} \lambda_{i,1,k} |ik\rangle$

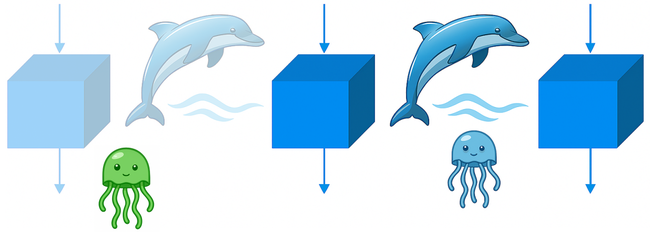


- When party C makes ***computational basis measurement*** the remaining two parties are projected to $\sum_{i,j} \lambda_{i,j,0} |ij\rangle$ and $\sum_{i,j} \lambda_{i,j,1} |ij\rangle$.

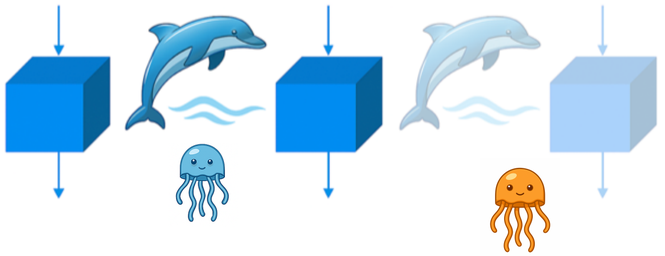


What we get?

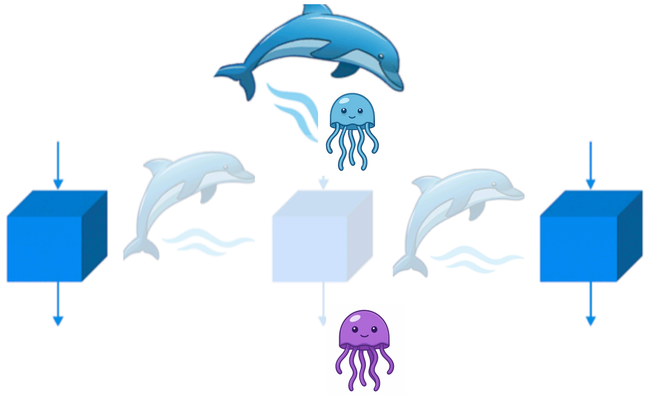
- When party A makes some ***unknown measurement*** the remaining parties are projected to $\sum_{\tilde{j},\tilde{k}} \lambda_{0,\tilde{j},\tilde{k}} |\tilde{j}\tilde{k}\rangle$ and $\sum_{\tilde{j},\tilde{k}} \lambda_{1,\tilde{j},\tilde{k}} |\tilde{j}\tilde{k}\rangle$ in an ***unknown basis.***



- When party B makes some ***unknown measurement*** the remaining parties are projected to $\sum_{\tilde{j},\tilde{k}} \lambda_{\tilde{j},0,\tilde{k}} |\tilde{j}\tilde{k}\rangle$ and $\sum_{\tilde{j},\tilde{k}} \lambda_{\tilde{j},1,\tilde{k}} |\tilde{j}\tilde{k}\rangle$ in an ***unknown basis.***

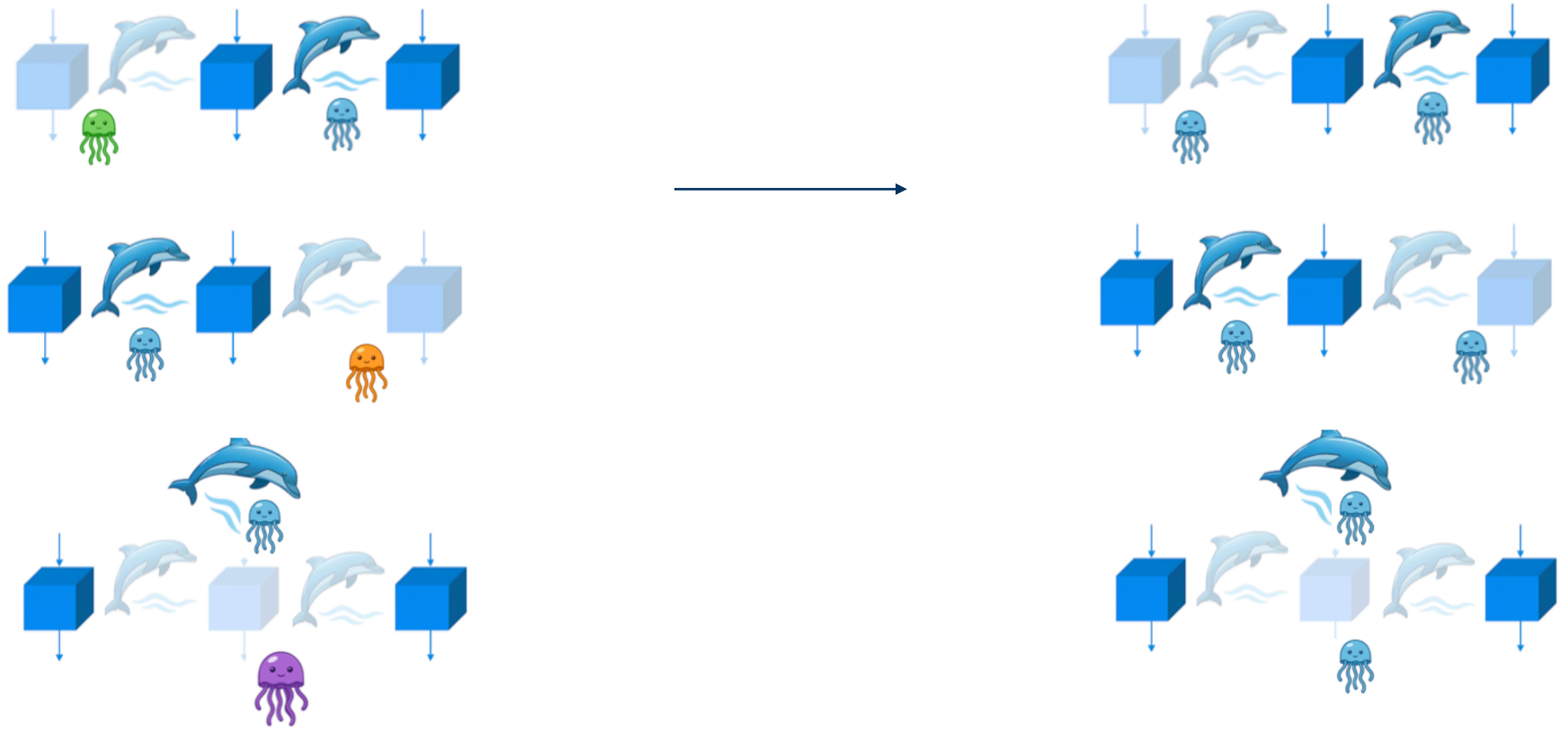


- When party B makes some ***unknown measurement*** the remaining par are projected to $\sum_{\tilde{j},\tilde{k}} \lambda_{\tilde{j},\tilde{k},0} |\tilde{j}\tilde{k}\rangle$ and $\sum_{\tilde{j},\tilde{k}} \lambda_{\tilde{j},\tilde{k},1} |\tilde{j}\tilde{k}\rangle$ in an ***unknown basis.***



Tripartite states

No jellyfish - marginal problem



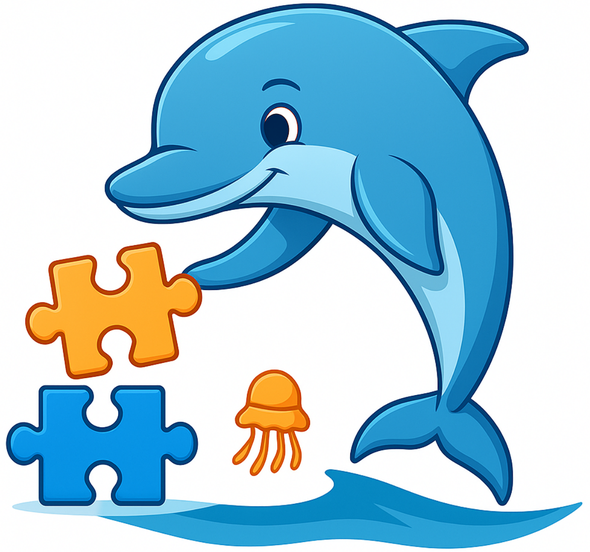
Neutralize jellyfish - relate projecting measurement and self-tested measurements

Tool 2: The measurement lemma

Let $|\psi\rangle$ be a bipartite state and $A_0^{(j)}, A_1^{(j)}, A_2^{(j)}$ dichotomic observables.

Suppose there exists unitaries U_j and U_k such that:

$$U_j \otimes U_k|\psi\rangle = |\psi_\theta\rangle \otimes |\xi\rangle \qquad U_j A_1^{(j)} U_j^\dagger = \sigma_X \otimes \mathbb{I}$$
$$U_j A_0^{(j)} U_j^\dagger = \sigma_Z \otimes \mathbb{I} \qquad U_j A_2^{(j)} U_j^\dagger = \sigma_Y \otimes A_Y$$



Suppose that

$$\langle\psi| A_0^{(j)} \otimes A^{(k)} |\psi\rangle = \alpha \qquad \langle\psi| A_2^{(j)} \otimes A^{(k)} |\psi\rangle = \gamma \sin 2\theta$$
$$\langle\psi| A_1^{(j)} \otimes A^{(k)} |\psi\rangle = \beta \sin 2\theta \quad \langle\psi| \mathbb{I}^{(j)} \otimes A^{(k)} |\psi\rangle = \alpha \cos 2\theta$$

where $\alpha^2 + \beta^2 + \gamma^2 = 1$, then we have

$$U_k A^{(k)} U_k^\dagger = \alpha\sigma_Z \otimes \mathbb{I} + \beta\sigma_X \otimes \mathbb{I} + \gamma\sigma_Y \otimes \bar{B}_Y$$

Allows us to fix the “measurement reference frame”

Tripartite states
Final statement

In the end, we are able to certify that

$$\Phi_{SWAP}\left(|\psi\rangle_{ABCP}\otimes|000>_{A'B'C'}\right)=|\psi\rangle_{A'B'C'}\otimes|junk_0\rangle_{ABCP}+|\psi^*\rangle_{A'B'C'}\otimes|junk_1\rangle_{ABCP}$$

$$\sum_{i,j,k}\lambda_{i,j,k}|ijk\rangle \qquad \sum_{i,j,k}\lambda^*_{i,j,k}|ijk\rangle$$

Multipartite states

Multipartite states

How to generalize the tripartite case

Consider the multipartite state:

$$|\Psi\rangle = \sum_{\vec{x}\in\{0,1\}^{\otimes N}} \lambda_{\vec{x}} \; |\vec{x}\rangle$$



Perform the sub-tests for all possible partitions.

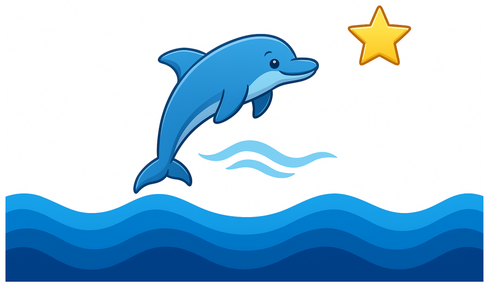


The total number of measurements (at least for 1 party) will grow exponentially.

Multipartite states

How to generalize the tripartite case

All the bases are the same → We can construct the isometry



$$\Phi[|\psi\rangle] = |\Psi\rangle \otimes |\mathrm{junk}_0\rangle + |\Psi^*\rangle \otimes |\mathrm{junk}_1\rangle$$

We can self-test multipartite entangled state of qubits which is equivalent to its complex conjugate.

For any state which is not equivalent to its complex conjugate, we exhibit a correlation that singles out the state up to complex conjugation.

Summary

Summary

Self-testing: way of certifying the state and the measurements only from the correlations → device-independent.



First proof that every qubit entangled state can be self-tested

